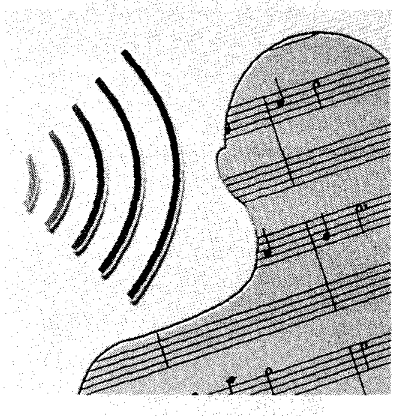

The Lambdoma Matrix and Harmonic Intervals

The Physiological and Psychological Effects on Human Adaptation from Combining Math and Music

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Understanding the musical/mathematical wisdom encoded within the Lambdoma matrix can lead to the invention of new devices and procedures for the engineering and medical communities. For example, the Lambdoma matrix, with its ability to define only harmonic interval sounds, suggests a new method to test the effects of harmonic music on human adaptation. By becoming familiar with the structure of the Lambdoma matrix, we learn that this combining of music with mathematics becomes a key for unlocking doors for the further understanding of color, music, frequency, wavelength, shape, and angle.

The ancient history of the Lambdoma reveals that, in the past, rational numbers were directly linked to harmonic and subharmonic series in a matrix that can be translated into one-to-one correspondences with musical harmonic interval keynotes. When a decoding of whole-number ratios to musical harmonic frequencies (in Hertz) occurs, specific harmonic musical intervals may be audibly generated by computerized devices. The benefit of this translation allows many levels of insights to be unveiled to the practitioner, as well as to the listener. It is multiple translations, or encoding and decoding, that leads to exploring other areas that may be accessed according to the expertise and imagination of those who can investigate further the untold beauties of this harmonic musical matrix. The effects of Lambdoma harmonic matrix music are felt conceptually by those exposed to its own inspiring universal mystery and beauty of ordered ratios, geometries, and overtone and undertone harmonic series.

This ancient wisdom that has been rediscovered has led to an understanding on many levels, and to the creation of a purely harmonic musical instrument. On the effects of this music, one may ask, "How does the Lambdoma music satisfy the needs of well being, accomplishment, calmness, stimulation, discovery, fulfillment, awareness, and creativity in music?" One example: A nurse who considered herself a nonmusician sat down at the Lambdoma keyboard, chose her keynote and tempo, and played for 15 minutes in a state of childlike enjoyment and wonder at the harmonic intervals she created.

History of the Lambdoma Matrix

The first recorded diagram of the Lambdoma, illustrated in Fig. 1, from a translation of the book *Introduction to Arithmetic* by Nichomachus (c. 100 AD), shows a Lambda figure of whole-number ratios to outline how the principles of creation unfold out of the one [1]. In ancient Greece, this musical-mathematical matrix was as familiar as our multiplication table [2]. Musical intervals are inherent in this double-entry multiplication and division table, because whole-number ratios are formed that are harmonically linked to each other by a fundamental 1:1 ratio of a given frequency (Fig. 2). Could this matrix be the source of a forgotten understanding of the power of musical intervals? Pythagoras (500 BC), to whom the discovery of the Lambdoma was attributed (and of whom it is said that he spent 20 years of his youth in Egypt), is said to have calmed the "savage beast" with particular chords played on his lyre. Pythagoras is said to have color-coded the musical scale as C (red), D (orange), E (yellow), F (green), G (blue), A (indigo) and B (violet).

Is it possible that the Lambdoma was known in 1450 BC? A Lambdoma-type illustration of diagonals of squares was found on a bracelet from the New Kingdom Dynasty (1450 BC) in Egypt (Fig. 3). As an interpretation of this bracelet as a musical matrix, the following description is suggested. The bracelet included six columns down (which can be interpreted

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as subharmonics) and 20 or so rows across (harmonics). Beads (notes) were strung on gold wire (vibrating string). The beads were carnelian, lapis, and turquoise. This may represent a color-coding of the musical harmonics of the chord D (orange), A (indigo), and F (turquoise).

Centuries later, von Thimus devoted his life to writing two volumes (1868-76) on the Lambdoma [3]. Even though Georg Cantor (1829-1920) [4] did not mention the name Lambdoma, his array bears a one-to-one mathematical relationship to the Lambdoma ratios. Then followed Hans Kayser [5], Rudolf Haase [6], Levaire and Levy [2], Ernest McClain [7], Josclyn Godwin [8], Barbara Hero [9, 10], and Jonathan Goldman [11]; all of these authors adding their particular mathematical, musical, and philosophical interpretations to the Lambdoma concept.

Musical-Mathematical History of Ratios

Musical intervals based upon ratios may have been used in ancient times, and the Lambdoma matrix may have been integral to those musical systems. To take a more modern mathematical example as an illustration, the base of natural logarithms, "e" or "epsilon," can be calculated by adding the elements of the subharmonic column of the Lambdoma, from the first through the eighth subharmonic ($1 + 1/2 + 1/3 + ... 1/8 = 2.718$), and these ratios may be coded in appropriate audible frequencies and played, as can the other mathematical ratios referred to below.

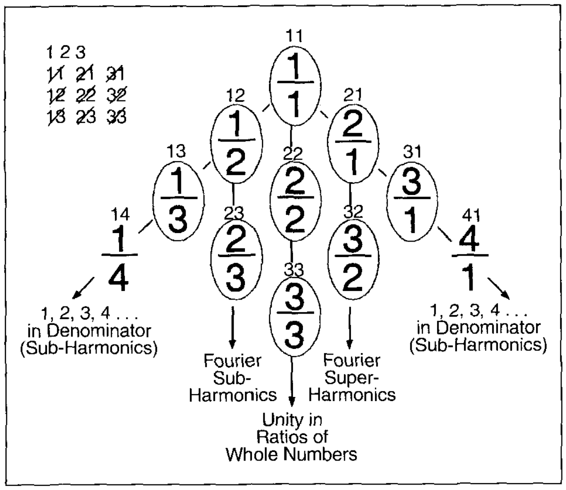
Numerical ratios existed in prehistoric times in Babylonia and Egypt, as both cultures used a scale of rational numbers to determine the value of phi (later referred to as the Fibonacci constant equal to 1.618) [12]. Phi may also be calculated from the Lambdoma matrix. In Archimedes' time "pi" was approximated as the ratio $3 + 1/7 = 22/7$.

The harmonic 3 and the subharmonic 7 may be found in the Lambdoma matrix. The musical notation of G (third harmonic) and D (seventh subharmonic) form a musical fifth. The ratio 22/7 becomes a G# (G sharp) at a frequency of 402 Hertz. Theon of Smyrna (130 AD) approximated the square root of 2 by the ratio of 7/5, an F# (F sharp) at a frequency of 358 Hertz in the Lambdoma matrix) [13].

Another connection with music and Egyptian mathematics is that the multiplication and division by two (diatonic) represents the mathematics of an octave in the language of music. A 16 by 16 Lambdoma matrix includes four harmonic octaves (rows) and four subharmonic octaves (columns). The Egyptian method of multiplication, used from at least 200 BC, was made up of two columns. One column was multiplied by two. The other column was divided by two. This is similar to our present-day binary system [13].

Helmholst (1821-1894) has also paired ratios with intervals, all of which are available on the Lambdoma matrix. For example: Unison C (1:1), Second D (8:9), Supersecond D+ (7:8), Subminor Third Eb– (6:7), Minor Third Eb (5:6), Major Third E (4:5), Supermajor Third E+ (7:9), Fourth F (3:4), Subminor Fifth Gb- (5:7), Fifth G (2:3), Minor Sixth Ab (5:8), Major Sixth A (3:5), Subminor Seventh Bb– (4:7), Minor Seventh Bb (5:9), Octave C (1:2) [14]. However, Helmholtz is using the string length intervals rather than frequency intervals, which are in inverse relationship.

The mathematical Farey series of gear ratios [15] (Fig. 4), bears a one-to-one correspondence to the Lambdoma matrix.



1. The basic Lambdoma matrix, named after the Greek letter "lambda." The basic V shape is the first written indication of the Lambdoma, which was called a "Lambdoid" in a footnote in one of the books written by Nichomachus in 100 AD. This work was translated by Iamblicus a century later. Modern notation is added to the original ratios, with references to Fourier, supraharmonics (musical overtones), and subharmonics (musical undertones).

This correspondence was brought to our attention by the musicologist Ervin Wilson [16]. Since the Farey series is generated by the Diaphantine equation, it seems likely that the Lambdoma may be generated by the same equation if the rational numbers are reduced to their least common denominators.

Description of the Lambdoma Matrix of Whole-Number Ratios

The diagonal of the Lambdoma matrix (Fig. 2) is always equivalent to a 1:1 ratio. The diagonal separates (superharmonic ratios) $n > 1$ moving from left to right from (subharmonic ratios) $n < 1$ moving from the top down. Musically, the harmonic series becomes higher in pitch as it moves horizontally across a row. On a stringed instrument, the higher pitches are created as the string is "stopped" shorter and shorter. Musically, the subharmonic series becomes lower in pitch as it moves vertically down a column. Since the Lambdoma matrix is pictured on the fourth quadrant of an x and y Cartesian coordinate system, each undertone column commences at a higher pitch than its previous neighbor, while each overtone row commences at a lower pitch than its neighbor above. Both the harmonic and subharmonic series form an anagram on the Lambdoma matrix. The diagonal 1:1 separates the upper triangular matrix from the lower triangular matrix. The Lambdoma matrix is also a "closed set" [17]. According to reference [22], a definition of a closed set is the following: "A subset F of R is 'closed' if it contains all of its limits. For example, intervals [a,b], (–infinity, b], and [a, infinity) are closed in R." Where F is a set, and where R is, "according to the Bolzana- Weirstrass theorem, any bounded infinite set of real numbers has a limit point in R." (Refer to Fig. 5.) The Lambdoma matrix also has psychophysiologic properties that might be labeled "quantum" in both the harmonic rows and the subharmonic columns. We find a similarity in the octave leaps from the level 1:1 to the level 2:1 or 1:2 to "quantum theory." According to Webster, it is the "theory that the emission or absorption of energy by atoms or molecules in not continuous but occurs in discrete amounts, each amount being called a quantum [18]." The second ratio, reading across from left to right in the first row, is 2:1, which is an oc-

C=256 Hertz

Fundamental	1:1 C	(2:1 C)	3:1 G	(4:1 C)	5:1 Eb	6:1 G	7:1 Bb	(8:1 C)	9:1 D	10:1 Eb	11:1 Gb	12:1 G	13:1 Ab	14:1 Bb	15:1 B	16:1 C
Octave 128	1:2 C	2:2 C	3:2 G	4:2 C	5:2 Eb	6:2 G	7:2 Bb	8:2 C	9:2 D	10:2 Eb	11:2 Gb	12:2 G	13:2 Ab	14:2 Bb	15:2 B	16:2 C
4th	1:3 F	2:3 F	3:3 C	4:3 F	5:3 Ab	6:3 C	7:3 D#	8:3 F	9:3 G	10:3 Ab	11:3 Bb	12:3 C	13:3 Db	14:3 D#	15:3 Eb	16:3 F
Octave 64	1:4 C	2:4 C	3:4 G	4:4 C	5:4 Eb	6:4 G	7:4 Bb	8:4 C	9:4 D	10:4 Eb	11:4 Gb	12:4 G	13:4 Ab	14:4 Bb	15:4 B	16:4 C
6th Minor	1:5 Ab	2:5 Ab	3:5 D#	4:5 Ab	5:5 C	6:5 D#	7:5 F#	8:5 Ab	9:5 Bb	10:5 C	11:5 D	12:5 D#	13:5 F	14:5 F#	15:5 G	16:5 Ab
4th	1:6 F	2:6 F	3:6 C	4:6 F	5:6 Ab	6:6 C	7:6 D#	8:6 F	9:6 G	10:6 Ab	11:6 Bb	12:6 C	13:6 Db	14:6 D#	15:6 Eb	16:6 F
2nd	1:7 D	2:7 D	3:7 A	4:7 D	5:7 F#	6:7 A	7:7 C	8:7 D	9:7 E	10:7 F#	11:7 G#	12:7 A	13:7 B	14:7 C	15:7 C#	16:7 D
Octave 32	[1:8 C]	[2:8 C]	3:8 G	[4:8 C]	5:8 Eb	6:8 G	7:8 Bb	[8:8 C]	[9:8 D	10:8 Eb	11:8 Gb	12:8 G	13:8 Ab	14:8 Bb	15:8 B	16:8 C]
7th Minor	1:9 Bb	2:9 Bb	3:9 F	4:9 Bb	5:9 D	6:9 F	7:9 G#	8:9 Bb	9:9 C	10:9 D	11:9 Eb	12:9 F	13:9 Gb	14:9 G#	15:9 A	16:9 Bb
6th Minor	1:10 Ab	2:10 Ab	3:10 D#	4:10 Ab	5:10 C	6:10 D#	7:10 F#	8:10 Ab	9:10 Bb	10:10 C	11:10 D	12:10 D#	13:10 F	14:10 F#	15:10 G	16:10 Ab
5th Diminished	1:11 Gb	2:11 Gb	3:11 Db	4:11 Gb	5:11 Bb	6:11 Db	7:11 E	8:11 Gb	9:11 Ab	10:11 Bb	11:11 C	12:11 Db	13:11 Eb	14:11 E	15:11 F	16:11 Gb
4th	1:12 F	2:12 F	3:12 C	4:12 F	5:12 Ab	6:12 C	7:12 D#	8:12 F	9:12 G	10:12 Ab	11:12 Bb	12:12 C	13:12 Db	14:12 D#	15:12 Eb	16:12 F
3rd Minor	1:13 Eb	2:13 Eb	3:13 B	4:13 Eb	5:13 G	6:13 B	7:13 C#	8:13 Eb	9:13 F#	10:13 G	11:13 A	12:13 B	13:13 C	14:13 C#	15:13 D	16:13 Eb
2nd	1:14 D	2:14 D	3:14 A	4:14 D	5:14 F#	6:14 A	7:14 C	8:14 D	9:14 E	10:14 F#	11:14 G#	12:14 A	13:14 B	14:14 C	15:14 C#	16:14 D
2nd Minor	1:15 C#	2:15 C#	3:15 G#	4:15 C#	5:15 F	6:15 G#	7:15 B	8:15 C#	9:15 D#	10:15 F	11:15 G	12:15 G#	13:15 A	14:15 B	15:15 C	16:15 C#
Octave 16	1:16 C	2:16 C	3:16 G	4:16 C	5:16 Eb	6:16 G	7:16 Bb	8:16 C	9:16 D	10:16 Eb	11:16 Gb	12:16 G	13:16 Ab	14:16 Bb	15:16 B	16:16 C
		OCTAVE	5th	OCTAVE	3rd	5 th	7th Minor	OCTAVE	2nd	3rd	4th Augmented	5th	6th Minor	7th Minor	7th	OCTAVE

2. The Lambdoma matrix. A 16 x 16 Lambdoma matrix (in the fourth quadrant of a Cartesian coordinate system) illustrating the one-to-one relationship of whole-number ratios to musical intervals. The x-axis of whole-number ratios indicates both a shortening of a string in wavelength, and a higher pitch in frequency, with whole-number-ratios as labeled. Note that an ascending musical scale commences at the eighth harmonic. The y axis indicates a "lengthening" of a string (hypothetical), a lower pitch, and a descending scale, also starting at the eighth subharmonic. The diagonal is always a 1:1 ratio and separates the lower triangular matrix of ratios 1, from the upper triangular matrix of ratios 1.

tave above the fundamental. This implies that a Lambdoma octave that contains eight steps has been negotiated in just one quantum leap.

The next ratio, 3:1, inserts a second level between 2:1, the first octave, and 4:1, the second octave. In the third octave of ratios, between 4:1 and 8:1, we find the ratios 5:1, 6:1, and 7:1.

It is only between the fourth octave and the fifth octave that a full eight-interval scale occurs. This is made clear when the fundamental ratio 1:1 is set at 32 Hz. Then, the full scale from 8:1 to 16:1 reads in *solfeggio* (voice exercises using scale: *do, re*,...) and with frequencies in Hz as *do* (256), *re* (288), *mi* (320), *fa* (352), *sol* (384), *la* (416), *ti* (448), *si* (480) and *do* (512). As alphanumeric musical notes, this nontraditional scale reads as <C, <D, <E, >F″, <G, <Ab, >Bb, <B, and <C, where the notation "<" indicates less than and ">" indicates greater than the standard 440-tuned Western scale. This will be described in more detail below.

Mathematically Deriving the Ratios that Generate the Lambdoma Matrix

One method of indicating an algorithm for deriving the ratios to generate the Lambdoma matrix is to set the limit of the *i*th row from 1 to 16, and the limit of the *j*th column from 1 to 16. Assign a selected audible frequency for the fundamental, 1:1. Next, divide this matrix by itself *i*/*j*. Usually, a fundamental in the middle octave is the best choice for being in a comfortable audible range, because the lower subharmonic columns may become subaudible in the division process. Setting the fundamental note C at 256 Hz, the lower limit interval column series becomes 32 Hz, while the higher limit row series becomes 4096 Hz. One of the features of this algorithm is that any audible fundamental frequency can be programmed into the matrix.

The formula for generating the Lambdoma matrix is as follows. The Lambdoma matrix has i,j entry:

$$\text{Matrix } (i,j) = f * j/i \,. \qquad (1)$$

The Subharmonic or Undertone Series

The subharmonic series, in music, is known as the undertone series [19]. Helmholtz, in his definitive book, *On the Sensations of Tone*, wrote: "Such deeper musical tones are called the harmonic under tones of the resonator. They are musical (over)tones whose periodic time is exactly 2, 3, 4, 5, and so on, times as great as that of the resonator. Thus, if the proper tone of the resonator is c″, it will be heard when a musical instrument sounds (the undertones) c′, f, c, Ab, F, D, C, and so on" [14]. c′ is the octave of middle c, c″ is the octave above middle c, and c‴ is two octaves above middle c.

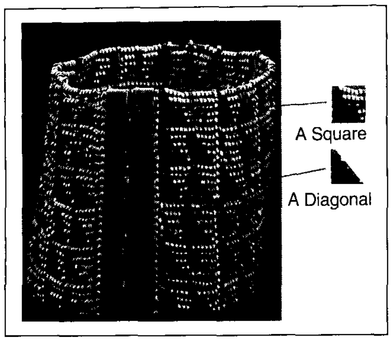
Helmholtz, writing about sympathetic vibrations and speaking of placing a wooden chip on a c′ string of a piano, writes: "The motion of the chip is greatest when one of the under tones of c′ is struck such as c, F, C, Ab, F, D, or C."

This suggests that sounding an undertone with the fundamental frequency will resonate a body physiologically more than an overtone or a partial will. This concept may have an application to the undertones either recorded from a software program or played by a Lambdoma musical instrument, in conjunction with a vibroacoustic device.

The tonal values of the undertones series in the Lambdoma matrix become c‴, c″, f″, c′, <a′, f′, d′, C, <B, <A, <G, F, <E, D, >C, C. These tonal values start at the first key-signature fundamental and become increasingly lower in pitch.

Differences of the Lambdoma Undertone and Overtone

If one counts the number of frequencies on the Lambdoma matrix between each subharmonic from the 1/1 to the 1/16, there is a difference between each that does not fall into the pattern of the harmonic series. The pattern of the harmonic series is easier to follow. Between each note in any Lambdoma reference octave, there is a well-defined whole-number frequency increment.



3. Was the Lambdoma matrix known over 3000 years ago? Diagonals of squares were found in a bracelet from the New Kingdom Dynasty in Egypt (1450 BC). The bracelet included six rows down and about 20 rows across. Beads were strung on gold wire, and the beads were carnelian, lapis, turquoise (orange, blue, and aqua). The bracelet's purpose is assumed to be protection of the body of the living and the mummy after death. This modified illustration is from the catalogue, *Splendors of Ancient Egypt*, The Detroit Institute of Art, 1997.

For example, in the middle C octave, one counts 32 whole-number frequencies between 256 (<C) and 288 (<D), 288 (<D) and 320 (<E), etc., until the end of the octave at 512. In the next higher Lambdoma octave, one counts 64 whole-number frequencies between each note in the scale. As one goes to higher octaves, the number doubles in each octave, and as one goes to lower octaves the number divides until we reach the fourth octave below 256, where there are only two frequencies between each note in that lower octave. All the frequencies are determined by the mathematical configuration of whole-number ratios in the Lambdoma matrix.

Other Whole-Number Systems with a One-to-One Relationship to the Lambdoma

Another system that bears a one-to-one relationship with the Lambdoma matrix is the Farey series [15], which may be generated by a Diaphantine equation below.

Diaphantine Equation (250 AD) Based Upon Whole-Number-Ratios:

$$bc - ad = 1 \qquad (2)$$

Farey Series:

$$\text{Epimore} = bc/ad \qquad (3)$$

If a/b and c/d are consecutive integers and $bc - ad = 1$, then bc/ad is the epimore.

$$\text{Mediant} = c/d \qquad (3)$$

If a/b, c/d and e/f are consecutive, then $(a + e)/(b + f) = c/d$ is the mediant.

Fractals and Cantor

Georg Cantor's transfinite array of rational numbers also bears a one-to-one relationship to the Lambdoma matrix of ratios [4], (Fig. 5).

The structure of much music is based upon fractals or "self-symmetry," claimed Dietrick Thomsen in an article in *Science News* [20]. The following is a quote from another article "Fractals: World of Non-integral Dimensions" in *Science News* [21]: "Sets and curves with the discordant dimensional behavior of fractals were introduced at the end of the 19th century by Georg Cantor and Karl Weierstrass." Their use at that time was limited to theoretical investigations in advanced mathematical analysis. Today, an additional application is its use in generating musical harmonics with the Lambdoma keyboard. Cantor proved the countability of ratios. He also was able to define derivatives of the Peano curve, as opposed to the Koch fractals, and this became a precursor to calculus. Another quote about fractals is the following: "Fractal behavior erupts whenever self-similarity forces the whole to be, in certain essential respects, the same as its parts" [21].

According to the Standard Mathematical Tables and Formulas [22]: "There is no universally agreed upon definition of fractal."

Lambdoma Matrix and Lambdoma Scales

The Lambdoma is a linear simultaneous multiplication (overtones) and division (undertones) harmonic matrix encompassing many keynote scales, with a constantly sounding fundamental ratio of 1:1 that creates the different harmonic musical intervals of the ratios. According to Webster [23], the definition of harmonic is: "In mathematics, designating a series of numbers whose reciprocals are in arithmetical progression." As illustrated in Fig. 2 this definition clearly corresponds to the undertone musical series in the Lambdoma matrix. The *Webster Dictionary*'s definition of harmonic reads: "In music, designating a tone whose rate of vibration is a precise multiple of that of a given fundamental tone." The latter statement clearly defines the overtone series in the Lambdoma matrix that multiplies each entry in each row by the fundamental tone. Since neither "undertone" nor "Lambdoma" are generally defined in most dictionaries, new definitions must be created for the Lambdoma matrix.

The Lambdoma scale, which is only the last part of the overtones series, consists of eight notes (one more than the standard musical scale), which are the upper part of the overtone series, from the eighth to the 16th harmonic. Why is one note missing in the diatonic scale? The scale was usually defined to be within an "octave," implying eight notes to the octave. The Lambdoma overtone scale is based upon whole-number ratios—unison 8:8 (*do*), 9:8 (*re*), 10:8 (*mi*), 11:8 (*fa*), 12:8 (*sol*), 13:8 (*la*), 14:8 (*ti*), 15:8 (*si*); and to the octave at 16:8 (*do*)—that are multiplied by any appropriate given audible frequency. According to Pythagorean thought, the diatonic scale originally was composed of ratios 1:1 (*do*), 9:8 (*re*), 81:64 (*mi*), 4:3 (*fa*), 3:2 (*sol*), 27:16 (*la*),

0/0
0/1 1/0
0/2 1/1 2/0
0/3 1/2 2/1 3/0
0/4 1/3 1/1 3/1 4/0
0/5 1/4 2/3 3/2 4/1 5/0
0/6 1/5 1/2 1/1 2/1 5/1 6/0
1/6 2/5 3/4 4/3 5/2 6/1
1/3 3/5 1/1 5/3 3/1
1/2 4/5 5/4 2/1
2/3 1/1 3/2
5/6 6/5
1/1

4. Applying the Diaphantine equation to the Lambdoma/Farey matrix. Add 0/1 and 1/0 to the Lambdoma matrix. Reduce the rationals. Apply the Diaphantine equation to row 6: $bc - ad = 1$; a/b and c/d are consecutive; ad/bc is "epimore"; a/b, c/d, e/f are consecutive; $(a + e)/(b + f) = c/d$ is the "mediant." The epimoria of a Lambdoma sequence of Order 6 is found by multiplying the numerator by the denominator of one ratio, and then the numerator by the denominator of the other ratio. The Farey series is 0/1, 1/6, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 5/6, 1/1, 6/5, 5/4, 4/3, 3/2, 5/3, 2/1, 5/2, 3/1, 4/1, 5/1, 6/1, 1/0. The related epimoria series is 1/0, 6/5, 5/4, 4/3, 6/5, 5/4, 6/5, 10/9, 9/8, 16/15, 25/24, 6/5, 6/5, 25/24, 16/15, 9/8, 10/9, 6/5, 5/4, 6/5, 4/3, 5/4, 6/5, 1/0.

243:128 (*ti*); and the octave at 2:1 (*do*). The tonic or 1:1 frequency could be set at any keynote, and, according to Guthrie, it might have been set at 192, which we would consider a G note [24]. The Lambdoma 1:1 entry may be set at any note, and all the other intervals within the matrix would be harmonically related because of its tight mathematical construct.

The Lambdoma and the Diatonic Scale

The diatonic is a recursive linear construct of only seven notes, and it is composed of major and minor modes that are identified as whole tones and half-tone steps in the scale. For further discussion of the diatonic, chromatic, and other scales see Valentinuzzi [25]. The diatonic scale, as we know it today, is not as truly harmonic as is the Lambdoma matrix. The chromatic scale of 12 tones and half tones is constructed on the basis of intervals related to one another according to an iterated irrational ratio defined by the 12th root of 2. The benefit of this construct is to create an equal tempered keyboard that allows a modulation from key to key. However, intervals of adjacent tones and half tones sounding together are always inherently dissonant in the diatonic system.

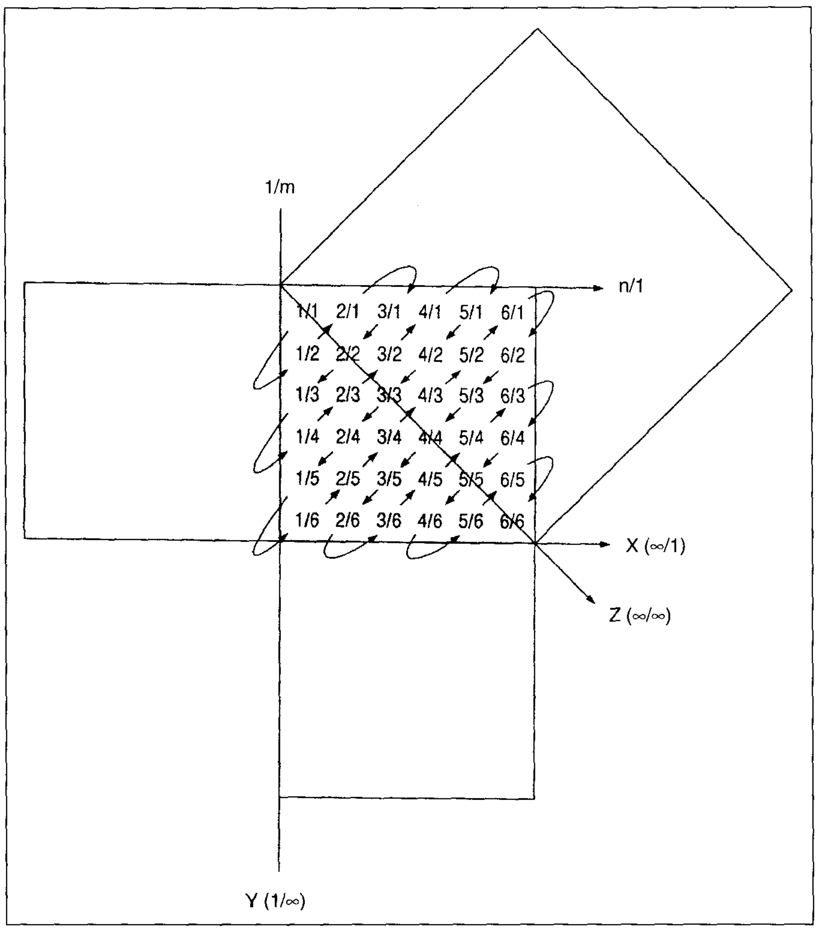
In the Lambdoma system, any two adjacent notes may be sounded simultaneously. These two notes are always inherently harmonious (hence physiologically pleasing to the listener) because of the Lambdoma's mathematical structure. The Lambdoma has a naturally equal interval system built into the overtones, which always have an equal number of frequencies between each entry. As one ascends to another octave, the equal intervals between each note become larger. As one descends in octaves in the overtone sequence, the equal intervals between each note become smaller.

With the advent of computer-generated frequencies such as those produced by the Lambdoma harmonic system, each musical key has its own characteristic flavor, and exact harmonic intervals may be generated from any audible keynote. There are harmonic microtonalities in the Lambdoma system, as well as harmonic overtones and subharmonics.

The notation "<" is used to indicate that the Lambdoma harmonic frequency is less than the diatonic note frequency. The notation ">" is used to indicate that the Lambdoma harmonic frequency is greater than the diatonic note frequency. We are using this notation throughout the article in order not to confuse the harmonic frequencies of the Lambdoma with the diatonic frequencies.

A Way of Modulating Keynotes from the Lambdoma Matrix

The Lambdoma matrix scales fall within the eighth to the 16th harmonic of eight keynotes (C, D, E, F#, G, Ab, Bb, B, and C). The frequencies of these eight keynotes, 32, 36, 40, 44, 48, 52, 56, 60 and 64 Hz, all compatible with the fundamental frequency of 32Hz (C), may be multiplied by 8 (the eighth harmonic) in order to form reference octaves that may be inserted in the fundamental 1:1 entry, so as to be completely compatible with our reference octave (Table 1). In Table 1, notice that the first column is composed of low-end audible frequencies. Each of these frequencies is separated by four Hertz. Each of these frequencies multiplied by eight would establish a fundamental that would form another Lambdoma matrix of eight intervals of a Lambdoma scale. In each 16 by 16 matrix thus formed, 256 different frequencies would be in total harmony with the whole. It is this harmonic factor of the parts relating perfectly to the whole that may be responsible for the unique cenesthesia response from listeners.



5. From a Lambdoma array (BC) to a Cantor array (2000 AD). The philosophical concept of the infinitely large and the infinitely small was common to many researchers of the Lambdoma matrix since Georg Cantor. This figure represents the x-axis (infinity/1) becoming infinitely higher in pitch (infinitely shorter in lengths of a stringed musical instrument), and the y-axis (1/infinity) becoming infinitely lower in pitch, while the diagonal stays in the same pitch. In order to include every ratio in his array, Cantor suggested a snake-like progression, as indicated. This construct was used as a mathematical device to prove that the rational numbers are a countable set. When this progression is made audible as music, it creates a pleasant roller-coaster effect to the listener.

In the Lambdoma system, it is important to have a reference octave as a constant so that one may have a standard of measurement. Our particular reference octave, obtained by multiplying 32 Hz by 8, is the following:

Lambdoma Reference Octave Overtone Scale of Fundamental 32 Hz (C): (Multiply 32 Hertz by the eighth through the 16th harmonics)

256 (<C), 288 (<D), 320 (<E), 352 (>F), 384 (<G), 416 (>Ab), 448 (>A), 480 (<B) and 512 (<C).

Comparison with Diatonic Scale Tuned to a Fixed Pitch of 440 Hz (A):

220 (A), 233 (A#), 247 (B), 262 (C), 277 (C#), 294 (D), 311 (D#), 330 (E), 349 (F), 370 (F#), 392 (G), 415 (G#) and 440 (A). In U.S. and in Europe A is sometimes 445.

Lambdoma Subharmonic Octave Scale of Fundamental 4096 Hz: (Divide 4096 Hz by the eighth through the 16th harmonics)

512 (<C), 455 (<A#), 410(>G#), 372(>F#), 341(>F), 315(>D#), 293(<D), 273(<C#) and 256(<C).

It is to be noted that the Lambdoma scales fall between the cracks of the piano keys. A Lambdoma keynote scale can be constructed in the frequency of any key, and it does not have to hold to a fixed tuning as would a piano. This is one of the advantages of using a computerized specific frequency, which enables a fine tuning of resonant bodies whether they be of organic or inorganic matter.

The Lambdoma Harmonic Keyboard

The Lambdoma harmonic keyboard, shown in Fig. 6, was created and produced as one result of the research and development into the theories formulated above. It is being used as an instrument to allow individuals to explore their own physical, mental, and emotional responses to the harmonic intervals [26].

Software-Hardware

Illustrated in Fig. 7 is a flow chart of Richard Lord and Robert Miller Foulkrod's conceptualization of how the theory of the Lambdoma matrix is embedded into the harmonic keyboard's hardware circuitry and software program. The current keyboard has a mahogany case and wooden keys. The keyboard itself contains a very small computer/controller running a key-scanning program. The keyboard is powered by an attached sound-generating computer, and the keyboard controls the stereo music signals generated by that computer. Each stereo interval-pair, which consists of line-level sine-wave output signals, is generated by the built-in, four-voice, sound chip of the attached computer. After booting, the computer speaks. The player is asked to input the desired duration of the interval-pairs that are to be generated, ranging from 1/16 of a second to four hours. The present 64-key keyboard plays any one chosen quadrant of the full 256-ratio Lambdoma matrix. The player is asked to input which 8-key by 8-key quadrant he or she wishes to play (from the full 16 by 16 matrix). Players are finally asked to enter their desired fundamental frequency (in Hertz) into the computer program (usually they use their own "keynote" frequency).

Table 1. Table of other reference octaves for a set of Lambdoma harmonics

Hertz	Note	8th Harmonic			through			16th Harmonic		
32	C	256	288	320	352	384	416	448	480	512
		c	d	<e	>f	g	<a	<b	b	c
36	D	288	324	360	396	432	468	504	540	576
		d	<e	>f	g	<a	<b	b	>c	d
40	Eb	320	360	400	440	480	520	560	600	640
		e	>f	>g	a	<b	c	<d	>d	e
44	F#	352	396	440	484	528	572	616	660	704
		>f	g	a	<b	c	<d	<e	e	>f
48	G	384	432	480	528	576	624	672	720	768
		g	<a	<b	c	d	<e	<f	>f	g
52	Ab	416	468	520	572	624	676	728	780	832
		<a	<b	c	d	<e	>f	<g	g	<a
56	Bb	448	504	560	616	672	728	784	840	896
		b	<c	<d	<e	<f	<g	g	<a	b
60	B	480	540	600	660	720	780	840	900	960
		b	>c	>d	e	>f	g	<a	<b	b

As each key is pressed or released, the internal key-scanning computer notices this action and transmits an ASCII message to the external sound-generating computer. See below for a description of the "BASIC Stamp" computer/controller.

The sound-generating computer stores these messages in a queue. The computer reads the messages in this queue and finds the ratio assigned to the key. The computer then produces the two different sine-waves and outputs them together as a stereo interval-pair. One sine-wave is the chosen fundamental frequency and the second sine-wave is that frequency multiplied by the Lambdoma matrix ratio for that key. The line-level outputs can be fed into any standard stereo sound system. The compiled "HiSoft Basic" sound-generating program runs automatically at computer start-up when installed on its internal hard drive. The program may be run from a floppy disk, also. The program may be used with any "Amiga" computer without modification of the computer.

Lambdoma Keyboard Scanner

The keyboard scanner, conceived and produced by Richard Lord [27], has been implemented using a Parallax Inc. (Rocklin, CA) BASIC Stamp module for economy and for ease of programming. Six of the I/O pins are used as outputs to a 74HC138 (demultiplexer IC) column scanner and a 74HC151 (multiplexer IC) row selector used to scan the 8 by 8 key-switch matrix. Signal diodes at each key-switch provide the needed multiple key isolation. The output of the 74HC151 is sensed by the seventh I/O pin. The microcontroller scans through the 64 key-switch locations by incrementing a register and writing it to the six output pins. For each count, the key-switch input is compared with a key-state table. The BASIC Stamp does not offer enough RAM storage to maintain this key-state table, but does offer sufficient space in EEPROM storage. Since the data EEPROM is rated for at least a million writes, failure is not likely within a lifetime of normal use of the keyboard. A change in state of any key causes the microcontroller to emit a key-press or key-release message at the serial output that has been assigned to the eighth I/O pin.

This mechanism is similar to that used in the standard MIDI musical keyboard interface. However, the BASIC Stamp is incapable of supporting the standard 62.5 Kb/s MIDI data rate, so the note messages are programmed in 7-bit ASCII readable characters at 2400 baud. For simplicity in the application program, the messages selected were "D n n <cr>" and "U nn <cr>" where "n n" are the decimal digits for the key number.

This encoding scheme is easily readable by a BASIC program running on the sound-generating computer, and it supports the ability to play multiple notes simultaneously. The data rate is sufficient for simple keyboard performance. In a future iteration, the microcontroller of the BASIC Stamp can be replaced with a new microcontroller, programmed in assembly language to support the standard MIDI specification for key events.

Encoding

Encoding Nutritive Elements of the Periodic Table into Music

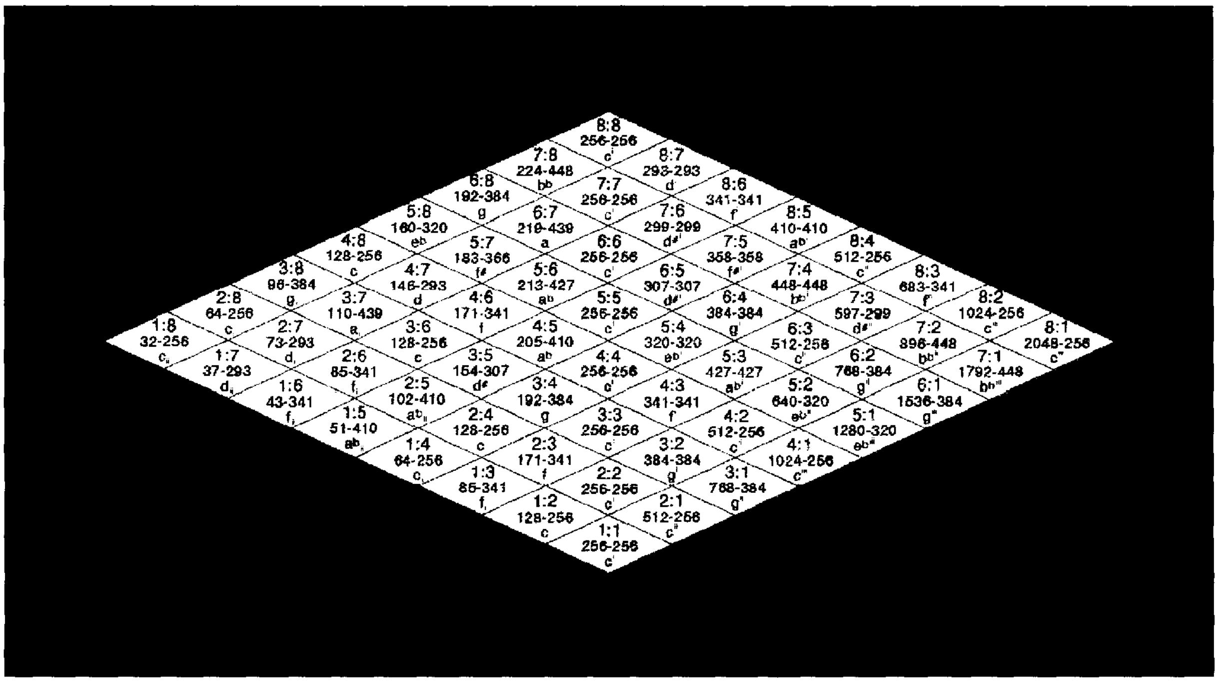
Since all foods contain ingredients that resonate at different frequencies, we decided to find the specific vibrations of certain minerals, as a supplemental form of food energy. The musical frequencies (*f*) of some of the minerals needed for our bodies were calculated based upon their atomic number. We transposed the atomic numbers by octave doubling to an audible range. Then, a corresponding interval was created by raising the reciprocal frequency (1/*f*) to an audible range in the same octave. Finally, a chord was created using the fundamental frequency, which was determined to be 256 Hz as the atomic number of hydrogen = 1 raised by doubling to the eighth octave. For example: Magnesium (Mg) = number 12 on the periodic table would have a musical note of (<G) at 384 Hz, and a reciprocal of (F) at 341 Hz. These two frequencies combined with the fundamental of 256 form a three-part musical chord, which can be played on a piano keyboard for some interesting results. An audio cassette of these harmonic intervals was produced, many of which often included the frequencies relative to the A, C, and E, which reflects the Fibonacci series [28].

Encoding Lissajous Figures into the Lambdoma Matrix

Cymatics is a form of witnessing the vibration of harmonic sounds on a flat plate. It has been found that harmonic sounds have characteristic shapes based upon their interval relationships to the fundamental of the Lambdoma system. At first, an oscilloscope, and later, a laser/scanner system, were used to present the shapes of the harmonic matrix intervals, which formed Lissajous shapes. As the music based upon the harmonic interval ratios was played, the Lissajous figures rotated, reflecting the x and y axes of the laser/scanner system [9]. This demonstration proved that two harmonic frequencies must sound together in order to create the polar-coordinate shapes of the Lissajous figures. The Lambdoma/Lissajous (Fig. 8) was created by one of Robert Miller Foulkrod's software programs, using the sine wave of the fundamental frequency and the cosine wave of the harmonic frequency on the two drivers of the laser/scanner system. An added application of such an ordered array of figures is to identify the whole-number ratios of mathematical knot theory. The effect upon individuals when they see the geometric shapes of the sounds that they are hearing is that it often opens another doorway to a cymatic experience.

Encoding Trigonometric Angles by Ratio into Music Platonic Solids

The computer-generated intervals of the Lambdoma keyboard enable one to choose any audible frequency for the fundamental 1:1 entry, so that a numerical frequency of any specific material or idea may be translated into a musical matrix vocabulary. Therefore, another method of encoding the

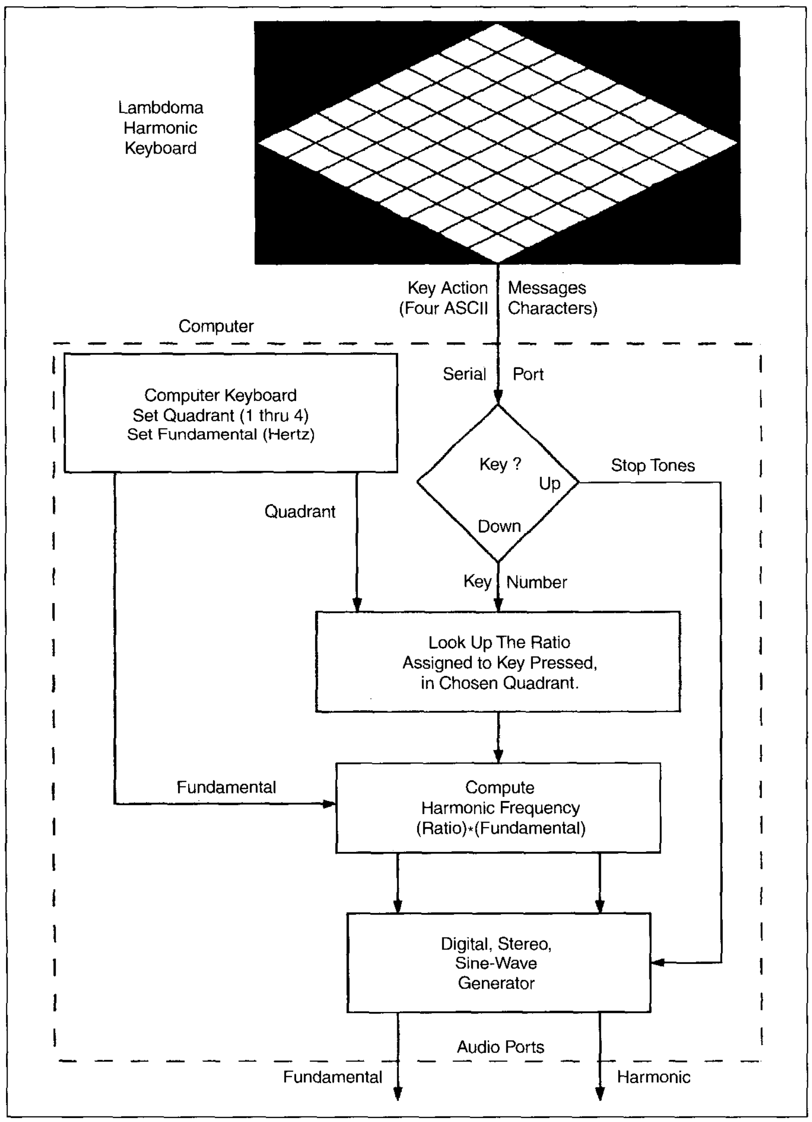


6. The Lambdoma harmonic keyboard. This figure shows the Lambdoma harmonic keyboard with a typical overlay, ready for an individual to choose a keynote. The overlay is of quadrant 1, with the ratios starting at 1:1 at the bottom apex. When running a finger along the keys of the lower row to the right apex, the harmonics sound within ascending octaves, from 256 to 2048 Hz. When running a finger along the keys of the lower row to the left apex, the subharmonics sound within descending octaves, from 256 to 32 Hz. This quadrant has 64 keys to press, as do the other 3 quadrants. Most individuals chose keynote frequencies one or two columns to the left of the fundamental, or at the fundamental or to the apex on the far right. Individuals are asked only to chose the sound that most appeals to them. When they have chosen their keynote frequencies, the specific frequency is noted and becomes the fundamental frequency for their own matrix on the Lambdoma keyboard. At each session, all four diamond shaped quadrants (subdivisions of the fourth Cartesian coordinate of Fig. 2) are played for about three minutes.

Lambdoma matrix is by translating angles into whole number ratios and then into frequencies to create an interval for any particular angle. By taking the arctangent angles of the Lambdoma matrix, and finding the "closest fit" angle in the matrix of whole-number ratios, musical frequencies related to the ratios may then be applied to the angles of the Platonic solids.

The angles of the Platonic solids can be translated into musical chords by this method. (Compare Table 2, "Figure Angles" to Fig. 2 "Lambdoma Table"). In Table 2, by taking the arctangent of each of the 256 ratios in the matrix, the closest fit of angle to frequency based on a fundamental of 256 Hz may be determined. The arctangent of 1:1 is a constant 45° along the diagonal. In order to seek a musical equivalent from a given angle found on the matrix, the frequency may be found between the ratio and the degree on the chart. To fine tune a given length angle with its particular frequency, a different fundamental would be used, and the corresponding frequencies and angles would be generated.

As an example, consider a tetrahedron. By using the encoding of angles described below, the tetrahedron angles can be shown to be equivalent to a note of <G. That is, the dihedral angle of this Platonic solid is 70°:31':44" [29]. Looking up this angle rounded off to 70° in Table 2, the Lambdoma Matrix of Ratios and Angles, we find that 70° is similar to an arctangent ratio of 14:5 or 11:4. These ratios correspond to <Gb at 717 Hz, or >F at 704 Hz when we use 256 Hz as our all pervading fundamental frequency (Fig. 2). The ratio of 14:5 gives an angle of 70°:20':46", which is close to our given angle. We took the above angles and their reciprocals, 4:11 and 5:14, combined with the fundamental drone, 1:1, of the frequency, 256 Hz, to compose the music of the tetrahedron. From this encoding of angles from the Lambdoma matrix of ratios, we have created a musical cassette based upon the angles of the Platonic solids and demonstrated the torus-like shape of the tetrahedron by the use of the laser/scanner system.



7. Flow diagram of the Lambdoma harmonic keyboard.

Plimpton 322 Tablet

A surprising practical application for the encoding of angles came from discovering that there might be a significant correlation of an ancient mathematical cuniform tablet with the Chinese acupuncture meridian system. The Babylonian Plimpton 322 Tablet consists of 15 angles between 31 and 45 degrees [12]. The 15 angles from 31 to 45 sounded with a fundamental of 256 (1:1) encompass an ascending scale of microtonalities from <D# (154 Hz) of ratio 3:5 to <C (256). By creating 15 right triangles within the above degrees and taking the reciprocal degree of each triangle, a musical chord is formed by translating the angles to frequencies in the manner described above. We have created a musical cassette of the meridian system based upon these principles, using the Lambdoma Keyboard to generate the frequencies and chords. Such cassettes are expected to help specialists engaged in acupressure, especially if used with a vibroacoustic applicator.

For more information on vibroacoustic methods used at the National Institutes of Health (NIH) see reference [30]. The resulting mathematically harmonic interval frequencies of the Lambdoma have a marked effect on those volunteers who have thus far experimented with them.

Encoding Time Signature to Keynote by Ratio

We found that a more powerful effect of music on individual listeners results from a one-to-one correspondence of the ratios inherent in key signatures found in the Lambdoma matrix, paired with the ratios of time signatures. A method for composing music in this way from the Lambdoma matrix is to start with an audible fundamental (keynote) frequency (in Hz), and then to generate the whole-number har-

(Continued on page 69)

The Lambdoma Matrix

(Continued from page 68)

monic intervals based upon the ratios of the matrix. Whole number ratios may be used as musical scales, harmonies, rhythms, and timbres. Rhythms may be easily formed by assigning the time signatures of the ratios to be compatible with the particular keynote frequency. Example: A keynote of >E flat, at a frequency of 160 Hz (or doubling to the middle C octave of 320 Hz) has one of its ratios of 5/4 equivalent to a time signature of 5/4 (where a fourth note has five beats to a measure). One may note that church music is often written in the key of E flat. A waltz rhythm of 3/4 time (where a fourth note has three beats to the measure) might stimulate an action-oriented effect, if the keynote were to be <G of a ratio of 3/2 and a frequency of 384 Hz in the middle C octave. Since G is known as the dominant fifth in music, a powerful effect on listeners results from using identical time-signature ratios to the harmonic keynote ratios and their frequencies [31]. A cassette using the above method, titled *Grand Gallery Galaxy Sounds,* was composed, recorded, and played. The effects were laughter, and it appeared to put the audiences in a good mood.

Applications

Decoding the Lambdoma Matrix of Ratios into Any Audible Frequency

By translating this ancient musical-mathematical matrix into an audible musical vocabulary, this sinusoidal pure-harmonic-interval music becomes a tool to enhance physiological and psychological adaptation by allowing one to tune into consonant harmonies used in ancient Greece. Musical intervals are inherent in this double entry multiplication and division table, because whole-number ratios are formed, which are harmonically linked to each other by a fundamental 1:1 of any given frequency (Fig. 9). It is this coupling of an audible fundamental frequency with consonant intervalic ratios that seems to aid individuals in experiencing a transcendental reality that creates profound states of well-being on hearing such pure harmonic intervals.

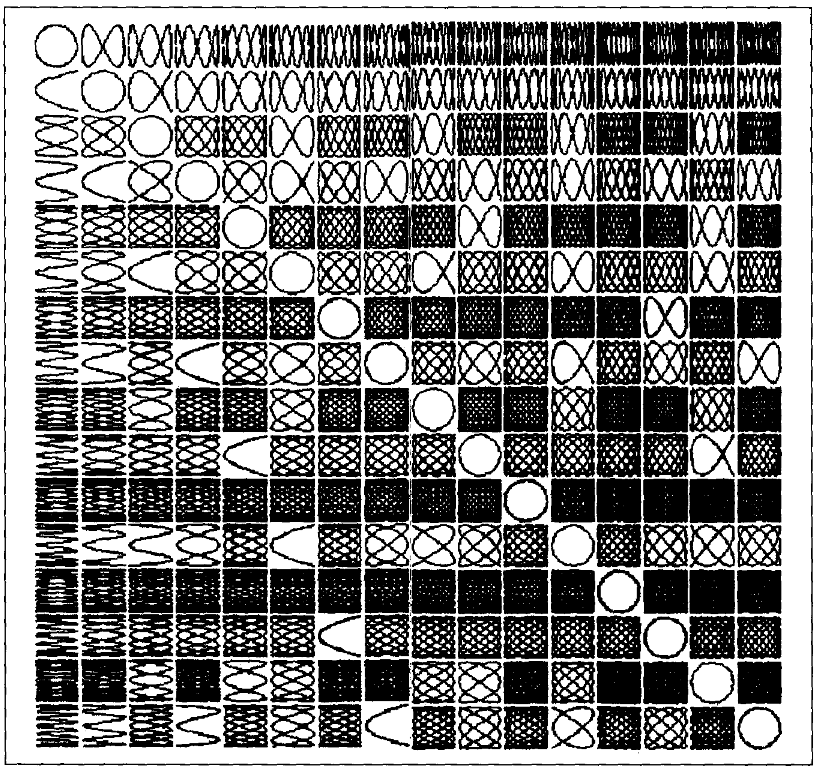
Octave Ratio (2:1) of the King's Chamber in the Great Pyramid in Egypt

The resonance of an enclosed space may be measured and then translated into musical harmonic proportions, which may help human adaptation to surrounding spaces. For example, the dimensions of the King's Chamber in the Great Pyramid in Egypt are 34 feet long, 17 feet wide, and 19 feet high; ratios of 2:1, 1:1 and 9:5 (Table 3) [32]. The frequencies of 34, 17, and 19 feet are 1130/34 = 33 Hz (>C lower octave), 1130/17 = 66 Hz (>C musical note), and 1130/19 = 59Hz (<B musical note). The resulting musical chord, which relates to the King's Chamber of the Great Pyramid, could be the musical notes >C and <B including all the harmonics of those frequencies of 33 Hz, 59 Hz, and 66 Hz, within the frameworks of >C and <B octaves.

In Table 3, a color-coding of each of the 256 entries would indicate the relation of exact proportions of a room to complementary colors. The musical note at 33 Hz is C at 264 Hz in the middle C octave. Its color is red, its ratio is 1:1, with doublings to 16:16. When the matrix is color-coded; rainbow rays of red, orange, yellow, green, blue, and purple would be seen radiating from the 1:1 apex at the upper left in the matrix. Complementary colors would be red/green, orange/blue, and yellow/purple. The complementary musical notes would be C/F, D/G, and E/B. Complementary ratios would be 1:1/2:3, 9:8/3:4, and 5:4/8:9. Complementary frequencies would be 33/22, 37/25, and 41/29. Complementary wavelengths would be 34/51, 30/46, and 27/39.

We have found in workshops that, when individuals hear a recording that we made in the King's Chamber in 1985, some individuals clearly envision the place, while others have emotions triggered and are profoundly affected by the music. Improvisations with flutes and voices accompanied the droning of one of the resonant frequencies of the chamber on the lowest string of a cello.

The fundamental continuous drone (1:1 ratio) probably causes the most profound changes in individuals. The drone is an important ingredient in all of the harmonic intervals created by the Lambdoma system, because when the fundamental frequency sounds with each harmonic or subharmonic, other harmonic intervals are always formed.



8. Lissajous figures coded to the Lambdoma matrix. The significance of this diagram is that the Lissajous figures are in an ordered sequence, so that it is easy to identify each figure with its specific frequency ratio with respect to the fundamental. This particular diagram was programed using sine and cosine algorithms with a one-to-one relationship to the whole-numbered ratios in the Lambdoma matrix. These Lissajous figures are also seen by the individuals (with the laser/scanner attachment) as they play each of the intervals with the Lambdoma harmonic keyboard.

Anecdotal Effects: Private Sessions with Individuals Playing the Keyboard

Over 100 private sessions have occurred from individual consultations and workshops throughout the country from 1994 to the present. Keyboard skills are not necessary with the Lambdoma Keyboard. (For other research on the effects of keyboard skills, see reference [33].) A person untutored in music can operate the Lambdoma keyboard, and because of its mathematically precise intervals, all of the keys are harmonically intertwined with the fundamental frequency chosen. This is the reason that the players are told to "feel as though you are playing a piece of music for the universe, and allow your own fingers to be your kinesiology instruments." This guidance makes the players feel good and pleased, especially when they are given the recorded version of their own playing. (For research on cenesthesia states, see reference [34].) Because of our interest in seeing the shapes of harmonic whole-number-ratio intervals, almost all of the earliest music composed before the advent of the Lambdoma keyboard was geared to the Lissajous shapes of these intervals, as shown in Fig. 7. In the 1970s, two sine-wave stereo generators were first used with an oscilloscope to view Lissajous configurations created [28]. Later, a laser/scanner system was used to project the shapes of the intervals. When subjects saw the shapes of the intervals that they were playing on the keyboard, another dimension of interactive responses to the sounds as well as to the shapes occurred. One woman sang overtone intervals as she played.

The sound with sound recording of a performance is exciting, because one does not know which are the subject's overtones and which are those of the keyboard. Another individual sang as she played. There was one session in which another subject chose a subaudible frequency, which she felt in her physical body. There was another keyboard session in which an individual, lying on a vibroacoustic bed, felt vibrations near her liver as a Lambdoma harmonic recording was played. For this session, a recording composed of the frequency associated with the speed of sound through the liver was used as the audible fundamental drone. Individuals often chose the fundamental pitches that matched their voices. Women chose higher pitches in the $n > 1$ upper triangular matrix. Men most often chose pitches in the $n < 1$ lower triangular matrix. One woman was able to access a very keen mental state, in which she asked herself questions that were answered on a mental level. Several others experienced a lessening of facial, limb, and hip pain while playing their own chosen keynote on the Lambdoma keyboard. This technique may provide practitioners with another dimension of alternative healing, with clients listening to their own harmonic intervals of microtonal sounds.

Observed Physiological, Emotional and Mental Effects of Lambdoma Music

The following two cases are chosen from the many individuals who reacted positively to the Lambdoma matrix sounds. These responses were obtained before the development of the Lambdoma harmonic keyboard, although the principles are the same. The harmonic intervals were generated using a BASIC computer program created by Robert Miller Foulkrod. These tape-recorded sounds were generated with an IBM PC Jr. computer.

Case 1: "Since listening to a Lambdoma harmonic matrix tape, I have had new feelings of bodily excitement and awareness. Sometimes the feelings are sort of part orgasmic, and sometimes a sense of mental excitement, which I haven't experienced since I was 20, sometimes an over all happiness. I am sure these feelings are brought on by the tape, even though some of them come several days after use. Once, five hours after I used the tape, a prop plane went over the house and set up an astonishing resonance inside me."

Case 2: "The Lambdoma Chakra Meditation affected me in a very positive way at the Expo. Since then I have found the tape very useful in unwinding and grounding me, creating more peacefulness in my life."

Another recording that seemed to have a profound influence on listeners was "Vela X." This music contains a background from the pulsar called Vela X, whose sounds were captured by the Arecibo Observatory in Puerto Rico (1978). The pulsar's rhythm was similar to a regular heart beat. A descending

Table 2. Encoding ratio, frequency, and angle from a Lambdoma matrix

FREQUENCIES & ANGLES

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1/1	Freq.:	256	512	768	1024	1280	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096
	Angle:	45	63	72	76	79	81	82	83	84	84	85	85	86	86	86	86
/2	Freq.:	128	256	384	512	640	768	896	1024	1152	1280	1408	1536	1664	1792	1920	2048
	Angle:	27	45	56	63	68	72	74	76	77	79	80	81	81	82	82	83
/3	Freq.:	85	171	256	341	427	512	597	683	768	853	939	1024	1109	1195	1280	1365
	Angle:	13	34	45	53	59	63	67	69	72	73	75	76	77	78	79	79
/4	Freq.:	64	128	192	256	320	384	448	512	576	640	704	768	832	896	960	1024
	Angle:	14	27	37	45	51	56	60	63	66	68	70	72	73	74	75	76
/5	Freq.:	51	102	154	205	256	307	358	410	461	512	563	614	666	717	768	819
	Angle:	11	22	31	39	45	50	54	58	61	63	66	67	69	70	72	73
/6	Freq.:	43	85	128	171	213	256	299	341	384	427	469	512	555	597	640	683
	Angle:	9	18	27	34	40	45	49	53	56	59	61	63	65	67	68	69
/7	Freq.:	37	73	110	146	183	219	256	293	329	366	402	439	475	512	549	585
	Angle:	8	16	23	30	36	41	45	49	52	55	58	60	62	63	65	66
/8	Freq.:	32	64	96	128	160	192	224	256	288	320	352	384	416	448	480	512
	Angle:	7	14	21	27	32	37	41	45	48	51	54	56	58	60	62	63
/9	Freq.:	28	57	85	114	142	171	199	228	265	284	313	341	370	398	427	455
	Angle:	6	13	18	24	29	34	38	42	45	48	51	53	55	57	59	61
/10	Freq.:	26	51	77	102	128	154	179	205	230	256	282	307	333	358	384	410
	Angle:	6	11	17	22	27	31	35	39	42	45	48	50	52	54	56	58
/11	Freq.:	23	47	70	93	116	140	163	186	209	233	256	297	303	326	349	372
	Angle:	5	10	15	20	24	29	32	36	39	42	45	47	50	52	54	55
/12	Freq.:	21	43	64	85	107	128	149	171	192	213	235	256	277	299	320	341
	Angle:	5	9	14	18	23	27	30	34	37	40	43	45	47	49	51	53
/13	Freq.:	20	39	59	79	98	118	138	158	177	197	217	236	256	276	295	315
	Angle:	4	9	13	17	21	25	28	32	35	38	40	43	45	47	49	51
/14	Freq.:	18	37	55	73	91	110	128	146	165	183	201	219	238	256	274	293
	Angle:	4	8	12	16	20	23	27	30	33	36	38	41	43	45	47	49
/15	Freq.:	17	34	51	68	85	102	119	137	154	171	188	205	222	239	256	273
	Angle:	4	8	11	15	18	22	25	28	31	34	36	39	41	43	45	47
/16	Freq.:	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256
	Angle:	4	7	11	14	17	21	24	27	29	32	35	37	39	41	43	45

Lambdoma harmonic interval scale, using a sine-wave generator, was added to create a "sound with sound" effect. This tape was very calming to listeners, and when played in a desert in Giza, Egypt, in 1981, it seemed to put a small group of American tourists in a meditative state of mind.

Cenesthesia Effects of Room Resonances

The musical resonances reinforced by the geometric dimensions of the rooms where people live or work may have a pronounced effect upon people's abilities to function in optimal ways. If room dimensions are in a harmonic proportion to the ratios used in music, they may be considered as another dimension by which music affects human adaptation.

Mathematical ratios of architectural proportions were used by Vitruvius (1st century BC), who was one of the first known architects. One may apply certain physical laws to find the resonant frequency of a cubic or rectangular room. Let the length, width, and height of a room be x, y, and z variables. These may be measured linearly. In order to translate the dimensions of x, y, and z into frequencies in cycles per second (Hz), an inverse law applies, since wavelength and frequency are in inverse relationship to one another. The speed of sound in air (v) is approximately 1130 feet per second at room temperature (70°C), with the linear dimension of x feet, the formula for frequency, f is:

$$f = v/x = 1130/x \text{ Hz} \qquad (4)$$

Therefore, a rectangular room may resonate to a three-tone chord, or to the interval relationship of a two-tone chord if any two dimensions are doubled, or in an octave relationship, as in the King's Chamber as described above.

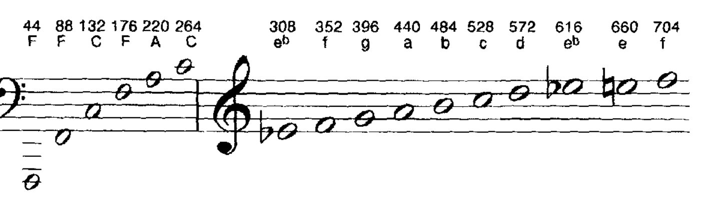
Observed Effects of Playing Lambdoma Music in a Drug Rehab Center

The recorded sound intervals based upon whole-number ratios had very calming effects in many instances when played to the residents of a drug rehabilitation center and to the inmates of a prison. The sounds released creativity in self-expression: visually in drawings and paintings, linguistically in poetry, and orally by allowing their voices to sound their emotions. Similar results were reported in the Tomatis method of listening to the music of Mozart and Gregorian chants for two hours, while the participants painted or talked [35]. We did note that when some residents selected their favorite popular music to play, they appeared to concentrate upon this music more than on their drawings. They seemed distracted by the rhythms of their musical choices.

The Lambdoma sound recordings that we played had minimal rhythm. (For the effects of rhythm on subjects, see reference [36].) An important factor in the effectiveness of these Lambdoma sounds seems to be that the residents had never heard such mathematically related sounds before. We conjecture that these sounds somehow entrained their brains so that creativity was released. These two musical structures (i.e., lack of rhythm and unfamiliar intervalic sounds) may have been factors that allowed the residents to be in a quasi-dream state, conducive to creativity. A controlled study is warranted to assess these conjectures. For the most part, they were hearing perfect intervals sounding at a very slow pace. The exception was the "Lambdoma Matrix" recording made at the Experimental Music Studio at the Massachusetts Institute of Technology in 1981. This recording was played when the residents seemed listless. The faster chords, with a steady ascending series of harmonic intervals that descended in an ordered sequence at each row, appeared to enliven the group.

X (Overtones)

Y (Undertones)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	352 f	704 f	1056 c	1408 f	1760 a	2112 c	2464 e^b	2816 f	3168 g	3520 a	3872 b	4224 c	4576 d	4928 e^b	5280 e	5632 f
2	176 f	352 f	528 c	704 f	880 a	1056 c	1232 e^b	1408 f	1584 g	1760 a	1936 b	2112 c	2288 d	2464 e^b	2640 e	2816 f
3	117 b^b	234 b^b	352 f	469 b^b	586 d	704 f	821 g	938 b^b	1056 c	1173 d	1290 c	1408 f	1525 f	1642 g	1760 a	1877 b^b
4	88 f	176 f	264 c	352 f	440 a	528 c	616 e^b	704 f	792 g	880 a	968 b	1056 c	1144 d	1232 e^b	1320 e	1408 f
5	70 d^b	140 d^b	211 a	281 d^b	352 f	422 a	492 b	563 d^b	633 e	704 f	774 g	844 a	915 a	985 b	1056 c	1126 d^b
6	58 b^b	117 b^b	176 f	236 b^b	293 d	352 f	410 g	469 b^b	528 c	586 d	645 c	704 f	762 f	821 g	880 a	938 b^b
7	50 g	100 g	150 d	201 g	251 b	301 d	352 f	402 g	452 a	502 b	553 c	603 d	653 e	704 f	754 f	804 g
8	44 f	88 f	132 c	176 f	220 a	264 c	308 e^b	352 f	396 g	440 a	484 b	528 c	572 d	616 e^b	660 e	704 f
9	39 d	78 d	117 a	156 d	195 g	234 a	273 c	312 d	352 f	3991 g	430 a	469 a	508 b	547 c	586 d	625 d
10	35 d^b	70 d^b	105 a	140 d^b	176 f	211 a	246 b	281 d^b	316 e	352 f	387 g	422 a	457 a	492 b	528 c	563 d^b
11	32 b	64 b	96 g	128 b	160 e	192 g	224 a	256 b	288 d	320 e	352 f	384 g	416 a	448 a	480 a	512 b
12	29 b^b	58 b^b	88 f	117 b^b	146 d	176 f	205 g	234 b^b	264 c	293 d	322 c	352 f	381 f	410 g	440 a	469 b^b
13	27 a	54 a	81 e	108 a	135 c	162 e	189 f	216 a	243 b	270 c	297 d	324 e	352 f	379 f	406 g	433 a
14	25 g	50 g	75 d	100 g	125 b	150 d	176 f	201 g	226 a	251 b	276 c	301 d	326 e	352 f	377 f	402 g
15	23 f	46 f	70 c	93 f	117 b^b	140 c	164 e	187 f	211 a	234 b^b	258 c	281 c	305 d	328 e	352 f	375 f
16	22 f	44 f	66 c	88 f	110 a	132 c	154 e^b	176 f	198 g	220 a	242 b	264 c	286 d	308 e^b	330 e	352 f



9. The harmonic series: a Lambdoma matrix coded to a fundamental frequency. This matrix is the same as Fig. 2 with a fundamental frequency of 352 Hz (the 11th harmonic of 32 Hz) applied to the fundamental 1:1 ratio. The harmonic series is shown on the staff below, starting with the eighth subharmonic row in the matrix, from 44 to 704 Hz. The harmonics ascend in frequency, whereas the subharmonics descend, and the diagonal maintains a constant frequency.

Table 3. Encoding ratio (top line of any element), frequency (in Hz), and wavelength (in feet) from a Lambdoma matrix

FREQUENCIES & DIMENSIONS

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1/1	Freq.:	33	66	99	132	165	198	231	264	297	330	363	396	429	462	495	528
	Feet :	34	17	11	9	7	6	5	4	4	3	3	3	3	2	2	2
/2	Freq.:	17	33	49	66	83	99	116	132	149	165	182	198	215	231	248	264
	Feet :	68	34	23	17	14	11	10	9	8	7	6	6	5	5	5	4
/3	Freq.:	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176
	Feet :	103	51	34	26	21	17	15	13	11	10	9	9	8	7	7	6
/4	Freq.:	8	17	25	33	41	49	58	66	74	83	91	99	107	116	124	132
	Feet :	137	68	46	34	27	23	20	17	15	14	12	11	11	10	9	9
/5	Freq.:	7	13	20	26	33	40	46	53	59	66	73	79	86	92	99	106
	Feet :	171	86	57	43	34	29	24	21	19	17	16	14	13	12	11	11
/6	Freq.:	6	11	17	22	28	33	39	44	49	55	60	66	71	77	83	88
	Feet :	205	103	68	51	41	34	29	26	23	21	19	17	16	15	14	13
/7	Freq.:	5	9	14	19	24	28	33	38	42	47	52	57	61	66	71	75
	Feet :	240	120	80	60	48	40	34	30	27	24	22	20	18	17	16	15
/8	Freq.:	4	8	12	17	21	25	29	33	37	41	45	49	54	58	62	66
	Feet :	274	137	91	68	55	46	39	34	30	27	25	23	21	20	18	17
/9	Freq.:	4	7	11	15	18	22	26	29	33	37	40	44	48	51	55	59
	Feet :	308	154	103	77	62	51	44	39	34	31	28	26	24	22	21	19
/10	Freq.:	3	7	10	13	17	20	23	26	30	33	36	40	43	46	49	53
	Feet :	342	171	114	86	68	57	49	43	38	34	31	29	26	24	23	21
/11	Freq.:	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48
	Feet :	377	188	126	94	75	63	54	47	42	38	34	31	29	27	25	24
/12	Freq.:	3	6	8	11	14	17	19	22	25	28	30	33	36	39	41	44
	Feet :	411	205	137	103	82	68	59	51	46	41	37	34	32	29	27	26
/13	Freq.:	3	5	8	10	13	15	18	20	23	25	28	30	33	36	38	41
	Feet :	445	223	148	111	89	74	64	56	49	45	40	37	34	32	30	28
/14	Freq.:	2	5	7	9	12	14	17	19	21	24	26	28	31	33	35	38
	Feet :	479	240	160	120	96	80	68	60	53	48	44	40	37	34	32	30
/15	Freq.:	2	4	7	9	11	13	15	18	20	22	24	26	29	31	33	35
	Feet :	514	257	171	128	103	86	73	64	57	51	47	43	40	37	34	32
/16	Freq.:	2	4	6	8	10	12	14	17	19	21	23	25	27	29	31	33
	Feet :	548	274	183	137	110	91	78	68	61	55	50	46	42	39	37	34

Discussion

We have dealt with the theoretical and with the genesis of a harmonic interval musical system. We postulate that from around 3000 BC to the present, there has been a continuity of one-to-one relationships of rational numerical systems to music. The hearing of a closed set of harmonically related intervals obviously has an effect on individuals, especially when they choose the continuously sounding fundamental frequency that most appeals to them. Could this appealing sound be based on their own anthropometric build and geometric dimensions? We found that often the measurement of their height corresponds to the musical frequency that they chose.

The advent of computers enabled us to choose any fundamental frequency for a whole-number-ratio music system, where an unlimited bank of harmonically and mathematically related frequencies is available. When more individuals hear and respond to the power of harmonic intervals of sound, musicians and others will have another tool at their disposal to bring harmonic relationships into their lives on many levels.

Results are not yet conclusive with the present protocols and number of samples, but they are very encouraging. More definitive understanding of physiological effects could be obtained with additional random sampling of individual Lambdoma harmonic keyboard players on different days and times of day.

Special biofeedback devices are needed to access the biophysical results of the effects of choosing fundamentals and playing the keyboard. A device could be designed to allow a finer tuning and choosing of the fundamental keynote, so as to be able to move the frequencies up or down several microtonalities. This might lead to even more definitive choices of keynotes. Otherwise, using the present fourth quadrant of the Lambdoma keyboard, which already has microtonalities available, might be a good choice for finer tuning. It has not yet been determined which octave ranges are the most effective for the choosing of the fundamental keynotes. Frequently, choices have been in the seven octaves between 32 Hz and 4096 Hz. The lowest keynotes are usually about 100 Hz, while many have chosen their keynotes within the middle C octave, 256 Hz to 512 Hz, and even two octaves above.

Some music was composed and recorded from the Lambdoma matrix based on the optimum speed of sound through different body organs, muscle, bone, blood, soft tissue, liver, and kidney. There has been no data on the effects of these specific frequencies, with the exception of using them in conjunction with a vibroacoustic mat.

In private sessions with the Lambdoma keyboard, we have observed that individuals using a "pillow speaker" to massage the intervals into their muscles feel the penetrating effects of Lambdoma sounds, and their pain seems to be relieved. Other music recordings were composed based upon the frequencies of minerals needed by the human body, as found in the periodic table of elements; no data on these effects have been reported.

Conclusion

Millenniums ago, in prehistoric times, the Babylonians and Egyptians were aware of the connection between tone and number. We are trying to reconstruct some of these laws of music by exploring the nature of the Lambdoma diagram of harmonic whole-number ratios. We hypothesize that the effects of this music, because of its well-defined mathematical construct, are that of emotional and physical harmony (cenesthesia). We strongly suspect that it is the continuously sounding fundamental pitch, chosen by the player, that is one of the most important ingredients as to the physiological and psychological effects of true harmonic music. When colors, shapes, angles, and sounds are all synthesized into one experience, the encoding of seemingly disparate numerical concepts from interdisciplinary fields into a musical vocabulary of ratios and related specific frequencies might lead to greater insights. These insights will illuminate each field, as well as enhance the significance of music itself in the human experience.

We may conclude that harmonic music is mathematics, and that harmonic mathematics is music, and that pure harmonic intervals combined with a well chosen, all-pervading fundamental frequency affect our physical, emotional, and mental states. We believe that the key to the effects of these harmonics upon human adaptation is based upon the whole-number-ratio interval structure of the Lambdoma combined with the fundamental drone, as well as the ability of the individual to select any audible frequency.

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She has been applying her knowledge of the visual art and musical-mathematics of the Lambdoma Matrix since the 1970s, publishing articles, books, and giving lectures and workshops at the University of Massachusetts in Amherst, the Institute of Technology in Rochester, NY, the Tomatis Center in Phoenix, the International Sound Colloquium in NH, the First International Conference on Music in Human Adaptation by the Virginia Polytechnic Institute and numerous other places including Hawaii and Bejing, China. She is Founder/Director of the ILRI (International Lambdoma Research Institute) in Wells, ME.

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