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Design in nature from Pythagoras to Helmholtz to the Cantor musical array

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Abstract

Our purpose is to illustrate how the laws of nature may be inherent in musical and mathematical systems. Pythagoras's musical system, based upon numerical ratios, was intrinsic to a natural order called "the music of the spheres". Helmholtz's musical system, based upon tonal laws, has an extremely important role to play in human adaptation. Georg Cantor, the metaphysical mathematician, developed an array of whole number ratios that follow the laws of microtonal musical harmonics and subharmonics.

While microtonal intervals play a limited role in Western music we find that they are intrinsic to a natural order. It is within the mathematical array that is called the Lambdoma Matrix where — visual patterns of color-coding, Lissajous figures coding and angles coding — mirror nature's laws. The Fibonacci series of ratios allows us to further explore laws of design and of nature. We have developed algorithms that generate the Cantor array in audible intervals that have not been heard before.

The engineering of intervallic harmonic stimuli may create a sensation of tonal values that apply to both external and internal harmonics. These sounds and their patterns stimulate a new kind of sonification and visual experience that indicate an extremely important role to play in the human sensation of tone and laws of design.

1 Introduction

Nature seems dependent upon an ordered set of harmonic ratios that may be directly related to the mathematics of music. Any number representing any phenomena in any scale from micron to light years may be translated into

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dimensions of wavelengths or musical periods of time using a mathematical matrix that converts ratios into real world phenomena. Nature uses visible rings in tree growth to show us time lines of centuries.

2 The Lambdoma Matrix of ratios

We have illustrated and described the array of ratios in the Lambdoma Matrix [Fig. 1] and in several present articles, [4], [5], and [6]. Ratios imply proportionality, based upon measurements that compare one whole number to another. A set of ordered ratios in a matrix forms a fixed framework, where a 1:1 entry in the matrix becomes the seed from which all other ratios flow and relate to each other. The matrix becomes an organism where all the parts are related to the whole. When you choose a new number to place in the 1:1 position of the matrix, that number becomes the new seed. This seed may be a frequency in

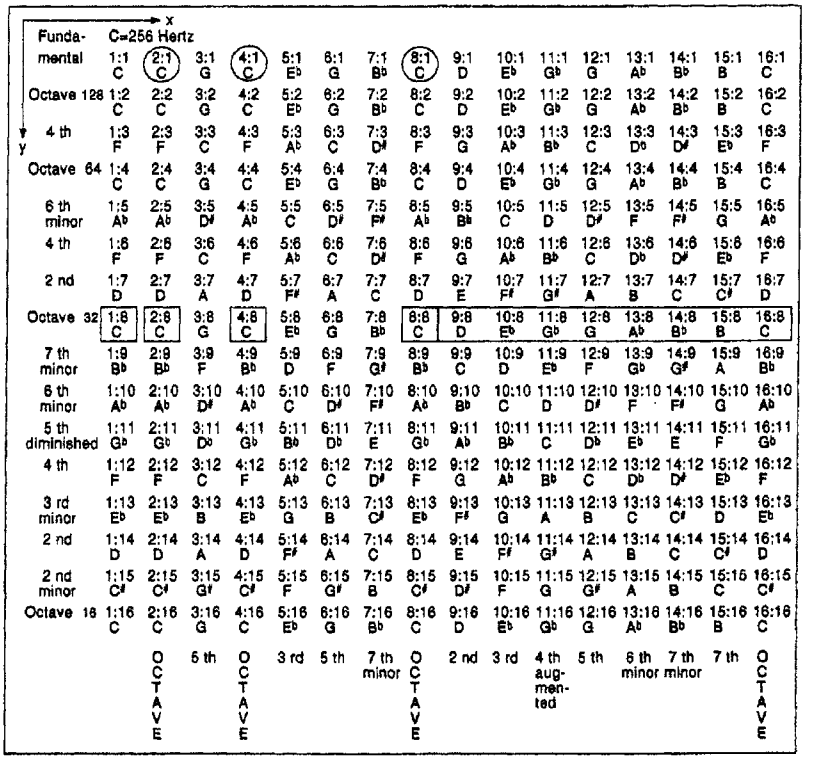

Figure 1: Example of the Lambdoma Matrix, showing the relationship of ratios to musical notation. Note that any audible frequency may be set at the 1:1 ratio in the matrix. The 1:1 frequency such as "Do" at 256 Hz as C will generate all the entries by multiplying and dividing by that frequency.

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Hertz (cycles per second) that becomes the fundamental of a matrix of harmonic musical sounds. The seed number may be a wavelength in feet that controls proportional lengths in architectural dimensions. The 1:1 seed value might represent a musical micron in biology, medicine or microelectronics. It might represent the periodic orbits of planets around the sun, or the number or density of an element in the periodic table.

3 The Pythagorean, Helmholtz and Cantor relationship

The factor, that ties the philosopher, Pythagoras [2], the physicist and musicologist Helmholtz [3] and the mathematician Cantor [4] together, is an ordered set of whole number ratios. Even though they were centuries apart, both Pythagoras and Cantor appear to have configured the same matrix of ratios. These matrices of ratios have a one-to-one relationship to each other, as well as a one-to-one relationship to music, as is illustrated in [Fig.s 1 – 3].

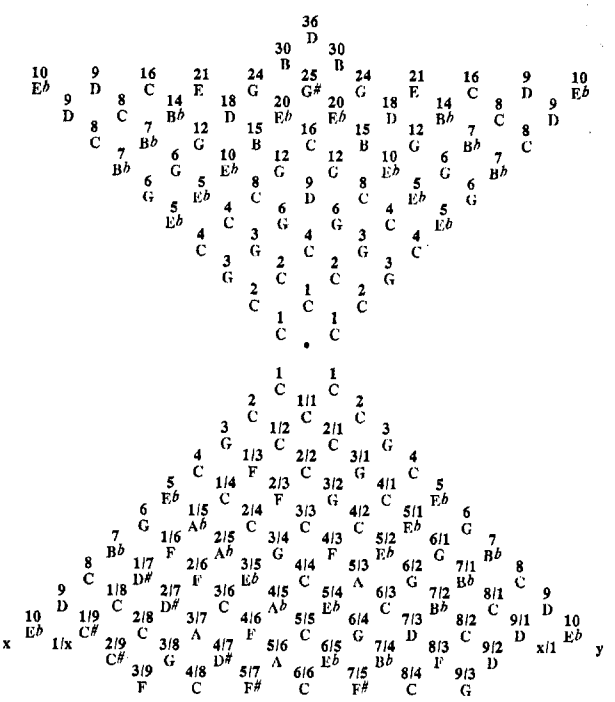

Figure 2: Pythagorean Chi (X) "The tetractys also forms the basis of the image of the cosmic soul, to whose structure in the form of a Chi (X) Plato refers in the 'Timaeus'. Inline with the law of the proportional division of the chord, the matrix of all earthly phenomena unfolds here as a network of coordinates of fractions and multiples" [1]. Note that the lower part of the Chi (X) is a 10 by 10 Lambdoma Matrix.

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8/10	8/9	8/8	8/7	8/6	8/5	8/4	8/3	8/2	8/1	8	16								Do
7/10	7/9	7/8	7/7	7/6	7/5	7/4	7/3	7/2	7/1	7	14	21							Mi
6/10	6/9	6/8	6/7	6/6	6/5	6/4	6/3	6/2	6/1	6	12	18	24	30	36				Re
5/10	5/9	5/8	5/7	5/6	5/5	5/4	5/3	5/2	5/1	5	10	15	20	25	30				Si
4/10	4/9	4/8	4/7	4/6	4/5	4/4	4/3	4/2	4/1	4	8	12	16	20	24				Sol
3/10	3/9	3/8	3/7	3/6	3/5	3/4	3/3	3/2	3/1	3	6	9	12	15	18	21			Mi
2/10	2/9	2/8	2/7	2/6	2/5	2/4	2/3	2/2	2/1	2	4	6	8	10	12	14	16		Do
1/x	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1/1	1	2	3	4	5	6	7	8	9	10 x
1/∞																			∞/1
1/10	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1/1	1/1	2/1	3/1	4/1	5/1	6/1	7/1	8/1	9/1	10/1
		1/16	1/14	1/12	1/10	1/8	1/6	1/4	1/2	1/2	2/2	3/2	4/2	5/2	6/2	7/2	8/2	9/2	10/2
			1/21	1/18	1/15	1/12	1/9	1/6	1/3	1/3	2/3	3/3	4/3	5/3	6/3	7/3	8/3	9/3	10/3
				1/24	1/20	1/16	1/12	1/8	1/4	1/4	2/4	3/4	4/4	5/4	6/4	7/4	8/4	9/4	10/4
				1/30	1/25	1/20	1/15	1/10	1/5	1/5	2/5	3/5	4/5	5/5	6/5	7/5	8/5	9/5	10/5
				1/36	1/30	1/24	1/18	1/12	1/6	1/6	2/6	3/6	4/6	5/6	6/6	7/6	8/6	9/6	10/6
							1/21	1/14	1/7	1/7	2/7	3/7	4/7	5/7	6/7	7/7	8/7	9/7	10/7
								1/16	1/8	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8	9/8	10/8
									1/9	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	9/9	10/9
									1/10	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	10/10
										La	La	Mi	La	Do	Mi	Fa	La	Si	Do

Figure 3: Pythagorean Chi (X), Lambdoma Matrix and Cantor array. The top quadrant to the right shows the upper part of the Pythagorean Chi (X) [Fig. 2]. The bottom quadrant to the right indicates the Lambdoma Matrix [Fig. 1], the Cantor array and the bottom part of the Chi (X).

4 Some visual patterns of sonification

Lissajous figures are formed by the interference of sine and cosine waves. The Lissajous shapes ordered within a 16 by 16 Lambdoma Matrix [Fig. 4] are defined by the specific ratios and create almost every shape from circle to infinity symbol to fish to intricate squares. In the same way ray-spirals may be generated based upon the Lambdoma ratios using the arctangent where the x-axis is divided by the y-axis [Fig. 5]. The latter might be a clue to the angels and shapes found especially in spiral galaxies, tornadoes and air and water vortices.

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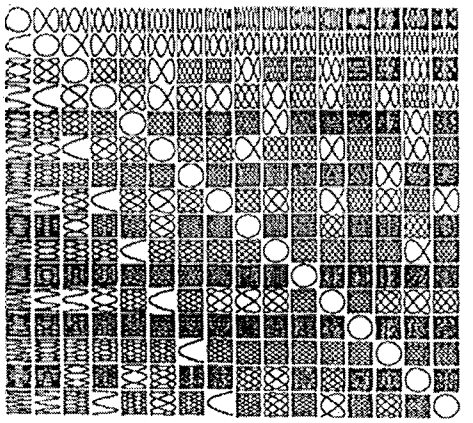

Figure 4: Lambdoma "Lissajous" coding.

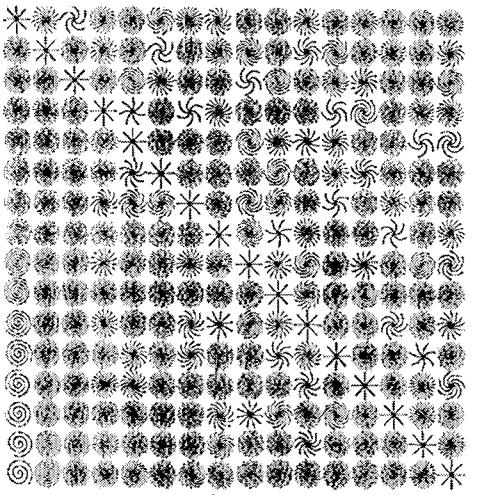

Figure 5: "Ray Spiral" coding.

5 Three dimensional Lambdoma structures

By taking the wavelengths of each ratio, structures are created that mirror nature's designs. For example, a matrix of four quadrants, built on each face of a cube creates a rhombic dodecahedron shape [Fig. 6]. A circular three-dimensional Lambdoma Matrix seems to create either a proportional architectural-like structure or a miniature mechanical type of tool [Fig. 7].

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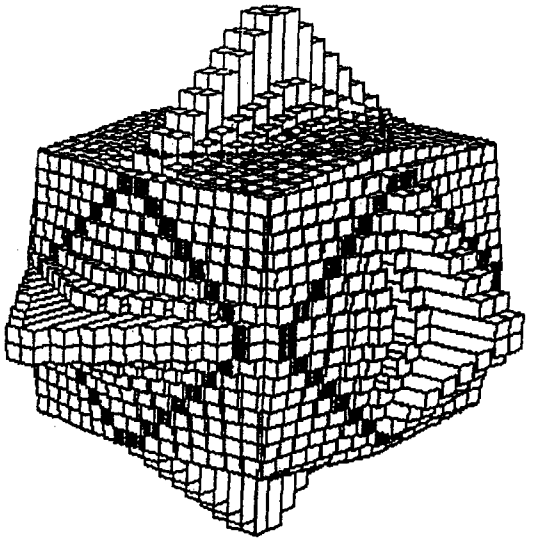

Figure 6: Lambdoma 3-D Rhombic Dodecahedron (Twenty-four quadrants of an 8 by 8 matrix)

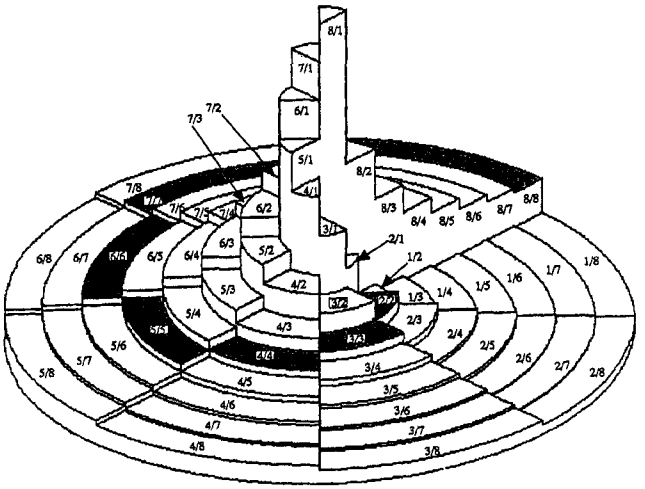

Figure 7: 3-D Mandala illustrating harmonics and subharmonics

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6 The algorithm that generates the harmonic sounds

Harmonics in music may be described as ratios with the numerator greater than the denominator. Subharmonics are in an inverse relationship to harmonics as the numerator is lessor than the denominator. The algorithm, for generating the ratios of the Lambdoma Matrix and its harmonics and subharmonics, follows:

$$n/d$$
$$\text{for } n = 1 \text{ to } 16$$
$$\text{and for } d = 1 - 16.$$

When any audible frequency is inserted in the 1:1 position, the entire matrix is defined by the ratios in the 16 by 16 matrix. In the same way angles may be generated using the arctangent where the x-axis is divided by the y-axis. This Lambdoma Matrix may be a clue to the configuration of angles of colors, feathers or scales.

7 Helmholtz relative to the Lambdoma

Helmholtz found an intensity of resonance embedded in ratios describing $^{\text{octaves}}$, 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, 6^{th} and 7^{th} intervals in musical notation. We added he 3^{rd} and 4^{th} columns that indicate the subharmonic and the harmonic series, in order to agree with the same ratios in the Lambdoma Matrix.

Table 1. Helmholtz table of intervals, notation and ratios

Intervals	Notation	Ratios/Frequencies Subharmonics	Ratios/Frequencies Harmonics
Unison	C	1:1 (256Hz) C	1:1 (256Hz) C
Second	D	8:9 (228Hz) Bb	9:8 (288Hz) D
Super 2nd	D+	7:8 (224Hz) A+	8:7 (293Hz) D+
Subminor 3rd	Eb-	6:7 (219Hz) A	7:6 (299Hz) Eb-
Minor 3rd	Eb	5:6 (213Hz) A-	6:5 (307Hz) Eb
Major 3rd	E	4:5 (205Hz) Ab	5:4 (320Hz) E
Supermajor 3rd	E+	7:9 (199Hz) G	9:7 (329Hz) E+
Fourth	F	3:4 (192Hz) G-	4:3 (341Hz) F

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Subminor 5th	Gb-	5:7 (183Hz) Gb	7:5 (358Hz) Gb
Fifth	G	2:3 (171Hz) F	3:2 (384Hz) G
Minor 6th	Ab	5:8 (160Hz) Eb	8:5 (410Hz) Ab
Major 6th	A	3:5 (154Hz) Eb-	5:3 (427Hz) A
Subminor 7th	Bb-	4:7 (146Hz) D+	7:4 (448Hz) Bb-
Minor 7th	Bb	5:9 (142Hz) D--	9:5 (461Hz) Bb
Octave	c	1:2 (128Hz) C	2:1 (512Hz) c

Conclusion

The Lambdoma Matrix of ordered ratios may be a key to open doors to phenomena that we know not of. Nature appears to use a matrix of this sort to generate life. The seed is, of course, whatever fundamental value is inserted in the 1:1 ratio from a star system to a blood cell to bacteria that describes its dimension in light years (time) or wavelength (space). Once you have a set of notations of musical frequencies, then it may be possible to discover hidden relationships by comparing seemingly unrelated phenomena.

References

[1] Roob A., *Alchemy and Mysticism*, the Hermetic Museum, Taschen: London, p. 98, 1997.
[2] Helmholtz, H., *On the Sensations of Tone*, Dover Publications: New York, p.125, 1954.
[3] Dauben, J. W., George Cantor: *His Mathematics and Philosophy of the Infinite*, Princeton University Press: Princeton, New Jersey, p. 52, 1979.
[4] Hero, B. F. & Foulkrod R.M., The Lambdoma Matrix and harmonic intervals, the physiological and psychological effects on human adaptation from combining math and music. *IEEE Engineering in Medicine and Biology Magazine*, **18(2)**, pp. 61-73.
[5] Hero, B., Barbara Hero: The Pythagorean system: the Tetractys, the X and the Lambdoma (Chapter 3). *Ambiguity and Music*, (ed). J. Haluska, *Seminar Mathematics and Music*, Bratislava, pp. 19-38.
[6] Hero, B. F & Foulkrod R. M., Integrative music of the Lambdoma. *Integrative Physiological and Behavioral Science, The Official Journal of the Pavlovian Society*, **35(3)**, pp. 224-232, 2000.